MODELING THE ACTIVE ZONE OF A WATER-MODERATED REACTOR WITH A VARIABLE NUMBER OF BOILING CHANNELS

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A scheme is proposed for the modeling of the dynamics of the active zone, with specified distributions for the heat-carrier flow rate and for the energy release, and the procedure involves the use of nonidentical boiling channels with quick-response fuel elements. As the initial condition, we use the possibility – demonstrated in this article – of a rigorous description (with a single channel) of a group of channels that are variously heated, but with a volume energy release similar for each.

The channels of an operating reactor exhibit various thermophysical conditions. They differ in terms of the magnitude of energy release, and the flow rate and heating of the heat carrier. Even in the case of ideal flow rate (identical heating in all channels), the channels exhibit diverse thermal inertia and react differently to perturbations in inlet enthalpy, power, etc.

If the heat carrier is boiling in some of the channels, its average density in such channels will differ substantially from those in which it is not boiling, and consequently these channels make different contributions both to the reactivity and to the change in the heat-carrier volume of contour I.

The active-zone model developed to investigate nonsteady regimes must ensure the required accuracy in the description of the changes in a number of averaged heat-carrier characteristics (averaged over a channel or a group of channels) (for example, the average volume density and the inlet enthalpy of the heat carrier, averaged over the cross section), as well as in the description of certain local characteristics (for example, the maximum variations in temperature or heat content at the channel outlet).

The capacity of a digital computer is inadequate to model each channel of an active zone in the study of nonsteady regimes for reactor operation, and a group of channels linked by some common indicator must thus be replaced by a single equivalent channel.

The resulting difficulties are associated with the need for a sufficiently precise description, simultaneously, of the average heat-release rate ($q_jL/G_j \neq \text{const}$) and inertia ($L/V_{0j} \neq \text{const}$).

We will demonstrate that on the basis of the familiar nonsteady distribution of enthalpy in a channel with the smallest static flow rate we can determine the precise distribution of enthalpy in another channel of the reactor, with the same volume release, but with a greater static flow rate for the heat carrier.

This enables us to average the channel parameters on the basis of only a single indicator, adhering strictly to the nonequivalence of the channels with respect to some other indicator. Obtaining the enthalpy distribution in the channel which is not modeled directly on the basis of the enthalpy distribution in a channel with another flow rate does not require the solution of differential equations.

The equations of continuity and heat balance for the j-th channel with a constant cross section and a heat flow constant over the length have the form

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$$S_j \frac{\partial \gamma_j}{\partial t} + \frac{\partial G_j}{\partial x} = 0,$$

$$S_j \frac{\partial (\gamma i)_j}{\partial t} + \frac{\partial (Gi)_j}{\partial x} = q_j(t) \tag{1}$$

in the assumption that the thermal inertia of the fuel element is small. Completing (1) with the equation of state

$$\gamma^{-1} = \begin{cases} (\gamma')^{-1}, & i < i', \\ (\gamma')^{-1} + \dfrac{(\gamma'')^{-1} - (\gamma')^{-1}}{r}(i - i'), & i > i', \end{cases} \tag{2}$$

we derive a system of equations with 3 unknowns $\gamma_j(x, t)$, $G_j(x, t)$, and $i_j(x, t)$. The quantities $G_{j\,in}(t)$, $i_{in}(t)$, and $q_j(t)$ are specified equations of time.

Let us examine (1) for $i < i'$.

We assume $q_j(t) = q_{0j}n(t)$, where q_{0j} is the static heat flow in the j-th channel; $G_j(x, t) = V_j(x, t)S_j\gamma_j = V_{0j}\xi(t)S_i\gamma_j$, where V_{0j} is the velocity of the heat carrier in the j-th channel.

From (1) we obtain

$$\frac{\partial i_j}{\partial t} + V_{0j}\xi(t)\frac{\partial i_j}{\partial x} = \frac{q_{0j}}{S_j}\frac{1}{\gamma_j}n(t). \tag{3}$$

Let us examine a group of k channels with similar volume heat-release rates $q_{j0}/S_j = \alpha$, $j = 1, 2, \ldots, k$.

Equation (3) is then rewritten in the form

$$\frac{\partial i_j}{\partial t} + V_{0j}\xi(t)\frac{\partial i_j}{\partial x} = \frac{\alpha}{\gamma'}n(t). \tag{4}$$

We will treat x as a function of time, satisfying (5):

$$\frac{dx_j}{dt} = V_{0j}\xi(t). \tag{5}$$

If (4) describes the time variation of enthalpy at each point of the j-th channel, the equation derived from (4) and (5) will evidently describe the time variation in enthalpy at a point moving along the channel at a speed commensurate with the velocity of the single-phase elements of the heat carrier in the channel.

Substituting (5) into (4) and solving the resulting equation, we find

$$i_j(t) = \frac{\alpha}{\gamma'}\int_{t_0}^{t} n(t)\,dt + i_j(t_0). \tag{6}$$

Let t_0 be the instant at which the particle enters the channel. Then $i_j(t_0) = i_{in}$, while (6) determines the enthalpy of the particles – which enter the channel with an enthalpy i_{in} at the instant t_0 – at any instant of time.

We note that (6) can be treated not only as the relationship for the determination of the enthalpies of the particles entering the channels simultaneously but also as the equation for the relative moments t_{1j} of which an equal enthalpy will be attained: $i_j(t_{1j}) = i_{sp}$. It is evident that t_{1j} is independent of the channel number.

From (5) we determine the coordinates corresponding to the particles entering the channels simultaneously and exhibiting identical enthalpies at a given constant of time in various channels:

$$x_j(t_1) = V_{0j}\int_{t_0}^{t_1} \xi(t)\,dt, \tag{7}$$

where t_1 may be arbitrary.

It follows from (7) that we have the relationship

$$\frac{x_j}{x_1} = \frac{V_{0j}}{V_{01}}, \quad j = 1, 2, \ldots, k. \tag{8}$$

Thus, having solved (4) for one j and having determined the enthalpy distribution over the length of the j-th channel at any instant of time, we know the enthalpy distribution in any channel of the group being examined. Indeed, the particles entering the channels simultaneously reach equal enthalpies, and this also simultaneously, but they cover paths in this case that are different, and these can be found from (8), without solution of additional differential equations.

We can demonstrate that an analogous conclusion can be drawn with regard to the two-phase segment.

Let us examine system (9):

$$\frac{\partial(\gamma_j i_j)}{\partial t}+\frac{\partial(V_j i_j \gamma_j)}{\partial x}=\frac{q_{0j}}{S_j}n(t),$$

$$\frac{\partial\gamma_j}{\partial t}+\frac{\partial V_j\gamma_j}{\partial x}=0,$$

where $V_j(x=0,\ t)=V_{0j}\xi(t)$ and $q_j(t)=q_{0j}n(t)$; γ and i are related by

$$\gamma=\gamma'(1-\varphi)+\gamma''\varphi,$$

$$\gamma i=(1-\varphi)\gamma' i'+\varphi\gamma'' i'',$$

from which, eliminating φ, we obtain

$$\gamma^{-1}=\frac{(\gamma')^{-1}r-i'[(\gamma'')^{-1}-(\gamma')^{-1}]}{r}+i\frac{(\gamma'')^{-1}-(\gamma')^{-1}}{r}.$$

We will subtract i from i* so that we satisfy $[(\gamma'')^{-1}-(\gamma')^{-1}]i_n/r$, where $i_n=i-i^*$.

This gives us

$$i^*=i'-r\frac{(\gamma')^{-1}}{(\gamma'')^{-1}-(\gamma')^{-1}}.$$

It is not difficult to see that replacement of the variables does not alter the form of the first equation in (9) when we consider the continuity equation. Indeed, the equation

$$\frac{\partial\{\gamma_j[i_{jn}+i^*]\}}{\partial t}+\frac{\partial\{\gamma_j V_j[i_{jn}+i^*]\}}{\partial x}=\frac{q_{0j}}{S_j}n(t)$$

reduces to the equation

$$\frac{\partial(\gamma_j i_{jn})}{\partial t}+\frac{\partial(V_j\gamma_j i_{jn})}{\partial x}=\frac{q_{0j}}{S_j}n(t). \tag{10}$$

However,

$$\gamma i_n=\left[\frac{(\gamma'')^{-1}-(\gamma')^{-1}}{r}\right]^{-1}=\text{const},$$

and (10) therefore assumes [1] the form

$$\frac{\partial V_j}{\partial x}=\frac{q_{0j}}{S_j}n(t)\left[\frac{(\gamma'')^{-1}-(\gamma')^{-1}}{r}\right]^{-1}. \tag{11}$$

Since we have chosen channels with identical volume energy-release rate for the group, we have

$$\frac{q_{0j}}{S_j}\frac{r}{(\gamma'')^{-1}-(\gamma')^{-1}}=\delta,$$

and the solution of (11) is found in the form

$$V_j(x,\ t)=\delta n(t)\left[x-h_{ej}(t)\right]+V_j\left[h_{ej}(t),\ t\right]=\delta n(t)\left[x-h_{ej}(t)\right]+V_{0j}\xi(t). \tag{12}$$

With consideration of (12) the continuity equation is written in the form

$$\frac{\partial\gamma_j}{\partial t}+\gamma_j\delta n(t)+\frac{\partial\gamma_j}{\partial x}\{V_{0j}\xi(t)+\delta n(t)\left[x-h_{ej}(t)\right]\}=0. \tag{13}$$

As in the previous case, we will assume that x is a function of time, determined by the equation

$$\frac{dx_j}{dt}=V_{0j}\xi(t)+\delta n(t)\left[x_j(t)-h_{ej}(t)\right]. \tag{14}$$

Moreover, we assume $\Psi_j(t)=\ln(\gamma_j(t)/\gamma')$.

Equation (13) then reduces to

$$\frac{d\Psi_j}{dt} = -\delta n(t), \tag{15}$$

whence

$$\Psi_j(t) = \delta \int_{t_{1j}}^{t_j} n(t)\,dt + \Psi_j(t_{1j}). \tag{16}$$

As was demonstrated above, the particles entering the channel simultaneously reach equal enthalpies at the same instant of time. Consequently, if t_{1j} denotes the instant at which the saturation enthalpy is attained in the j-th channel, we have $t_{1j} = t_1$ when $j = 1, 2, \ldots, k$ and

$$\Psi(t_1) = \ln \frac{\gamma(i')}{\gamma'} = 0.$$

Since dx/dt – determined from (14) – denotes the velocity of the heat carrier in the evaporation zone, we see from (16) that equal values of Ψ_j (and consequently, equal values of density and enthalpy) for particles entering the channels simultaneously, on motion through the evaporation zone, will also be attained simultaneously (although they will cover different paths in this case).

It remains to determine the relationship between the coordinates of the particles in the evaporation zone, these particles having entered the various channels simultaneously, and exhibiting equal enthalpies at a given instant of time.

Considering that $h_{ej}/h_{e1} = V_{0j}/V_{01}$, in the light of (8), let us transform (14) as follows:

$$\frac{dx_j}{dt} = V_{0j}\xi(t) + \delta n(t)\left[x_j(t) - \frac{h_{e1}(t)}{V_{01}}V_{0j}\right],$$

or

$$\frac{d\,\frac{x_j}{V_{0j}}}{dt} = \xi(t) + \delta n(t)\left[\frac{x_j(t)}{V_{0j}} - \frac{h_{e1}(t)}{V_{01}}\right]. \tag{17}$$

It is obvious that (17), solved for $x_j(t)/V_{0j}$, is identical for all the channels of the group and exhibits the same initial conditions:

$$\frac{x_j(t=0)}{V_{0j}} = \frac{x_1(t=0)}{V_{01}}.$$

Thus, at the one- and two-phase segments the coordinates of the particles entering the various channels simultaneously and exhibiting equal enthalpy (density) are associated by means of the relationship

$$\frac{x_j(t)}{V_{0j}} = \frac{x_1(t)}{V_{01}}.$$

Proceeding from the above, we can propose the following scheme for the modeling of the active zone:

1) all channels of the active zone are divided into several groups, with each group containing the channels with identical volume energy-release rates that are not dependent on the flow rate (the magnitude of heating);

2) from each group we select the channel with the smallest static flow rate, and for each we use a digital computer to solve the equations of continuity and heat balance;

3) from the derived enthalpy distributions over channel length for each instant of time we determine all required characteristics of any channel in the group under consideration, without direct modeling;

4) we calculate the characteristics at the outlet from the active zone by simple averaging of the appropriate parameters at the outlet from the channels, for a given instant of time.

NOTATION

L and S	are, respectively, the channel length and cross section;
V, G, γ, and i	are, respectively, the velocity, the flow rate, the density, and the enthalpy of the heat carrier;

q	is the heat flow per unit channel length;
x	is the coordinate along the channel;
t	is the time;
γ' and γ''	are the densities of the water and of the vapor at the saturation line;
i'	is the saturation enthalpy;
r	is the specific heat of vapor formation;
φ	is the volume vapor content;
h_e	is the length of the economizer zone.

Subscripts

0	pertains to the initial state;
in	denotes the inlet to the channel;
out	denotes the outlet from the channel;
j	is the channel number;
n	is a new variable;
sp	is a specified quantity.

LITERATURE CITED

1. E. P. Serov and B. P. Korol'kov, Dynamics of Processes in Heat- and Mass-Exchange Apparatus [in Russian], Énergiya, Moscow (1967).